Chapter review

1 Decide whether the following relations are functions or not.

a

x	−9	−5	0	1	3	5
y	−8	−6	−9	−1	0	5

b

x	−11	−1	−1	11
y	5	7	0	8

c

d

e

f $y = 3x + 2$

g $y = \sqrt{x} + 4$

h

i

j

(4, 16)

k

l

m

f

g

h
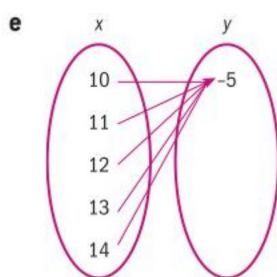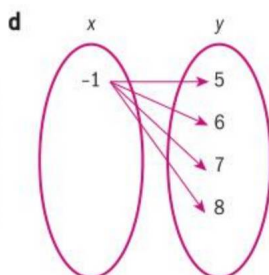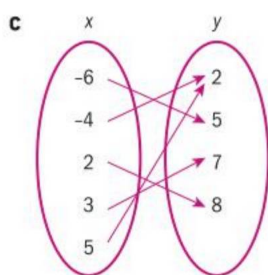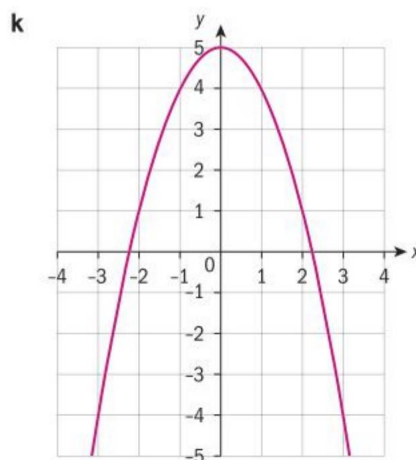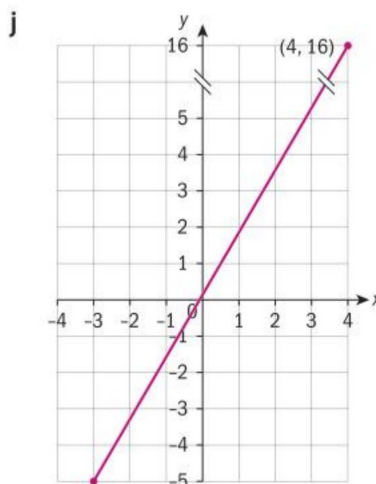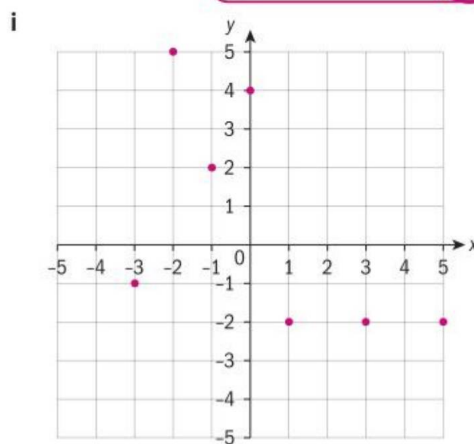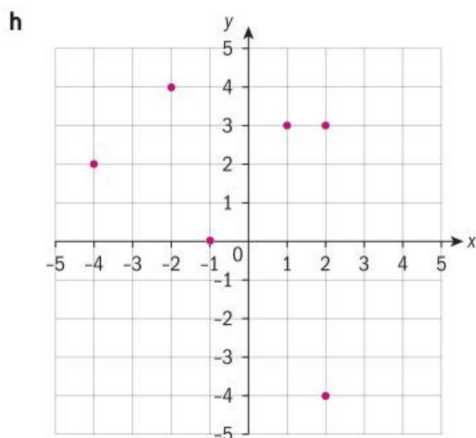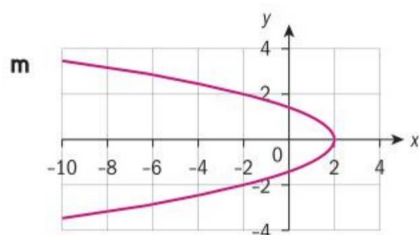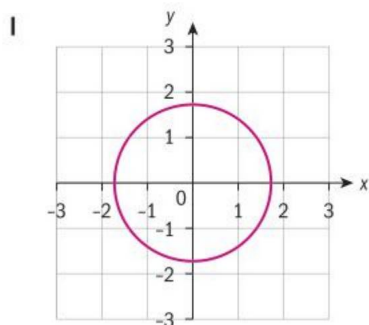
2 State the domain and range for the functions below:

a

x	−5	−1	0	1	4	9
y	−8	0	6	−1	9	1

b

c $\{(−8, 2), (−5, 3), (0, 2), (1, −2)\}$

d $y = -\dfrac{1}{2}x + 1$

e
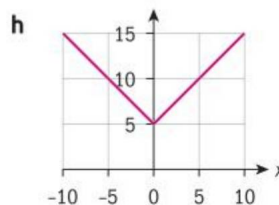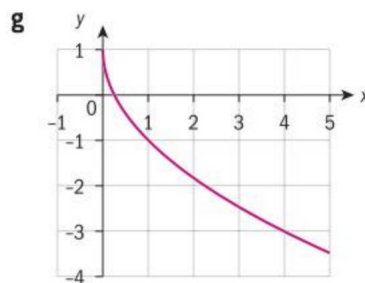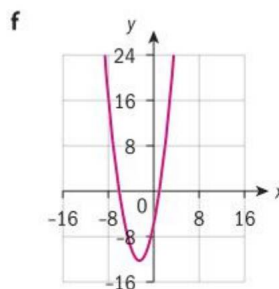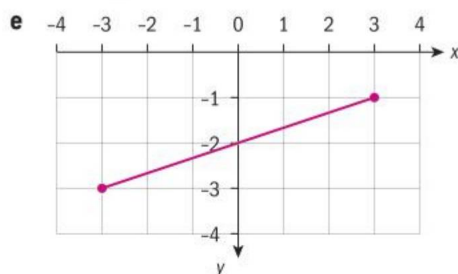
3 Use the functions $f(x) = x^2 − 6$, $g(x) = −2x$ and $h(x) = −4$ to evaluate:

a $f(3)$ **b** $f(−2)$ **c** $g(−6)$

d $f(1) + h(2)$ **e** $2f(0) − 2g(−1)$

f $h(0) \times f(−1)$ **g** $g^{-1}(−3)$ **h** $f(g(x))$

i $f \circ g^{-1}(x)$

4 Use your GDC to help you sketch the graphs of the following functions and state their domain and range:
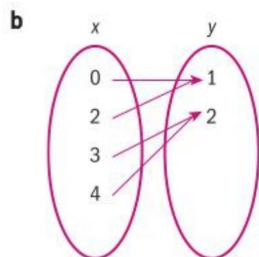
a $y = |x^3| − 2$ **b** $y = 2x^4 − 5x^3 + x − 2$

5 For the following pairs of functions, determine algebraically if they are inverses:

a $f(x) = −4x + 2$, $g(x) = -\dfrac{x − 2}{4}$

b $f(x) = \dfrac{1}{2}x − 4$, $g(x) = -\dfrac{x − 2}{4}$

c $f(x) = \dfrac{1}{2}x^2 + 4$, $g(x) = 2x + \dfrac{1}{4}$

d $f(x) = \dfrac{2x + 3}{3x − 1}$, $g(x) = \dfrac{3 + x}{3x − 2}$

6 The graph below shows $y = f(x)$ for $-3 \leq x \leq 2$.

a **i** Write down the value of $f(-3)$.
ii Write down the value of $f(2)$.

b Find the domain of f^{-1}.

c Sketch the graph of f^{-1}.

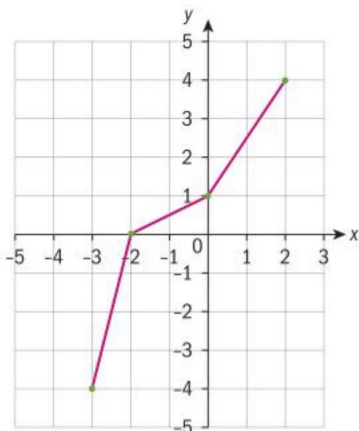**7** Let $f(x) = (x + 2)^3$. Let g be a function such that $(f \circ g)(x) = -8x^6$.

8 Let $f(x) = 2\sqrt{x} + x^2$. Let h be a function such that $h(16) = -2$. Find $(f \circ h^{-1})(-2)$.

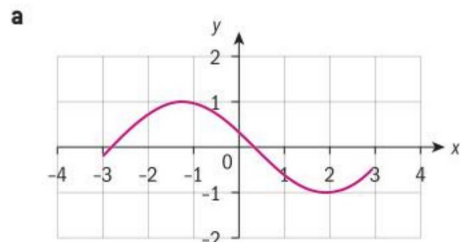**9** Show that $f(x) = -\dfrac{3}{x}$ is a self-inverse function.

Exam-style questions

10 **P1:** Find the range of the following functions.

a $f(x) = 5x + 1$, domain $\{x \in \mathbb{R}, -5 \leq x \leq 5\}$
(2 marks)

b $f(x) = 4 - 2x$, domain $\{x = -1, 0, 1, 2, 3, 4\}$ (2 marks)

c $f(x) = x^2$, domain $\{x \in \mathbb{R}, 0 \leq x \leq 10\}$
(2 marks)

d $f(x) = 250 - 12.5x$, domain $\{x \in \mathbb{R}, 0 \leq x \leq 10\}$ (2 marks)

11 **P2:** $f(x) = 4x - 2$, $x \in \mathbb{R}$ and $g(x) = x^2 - 8x + 15$, $x \in \mathbb{R}$.

a Find $f(-2)$. (2 marks)

b Find $g(-2)$. (2 marks)

c Find an expression for $f^{-1}(x)$.
(2 marks)

d Solve the equation $g(x) = 27$.
(4 marks)

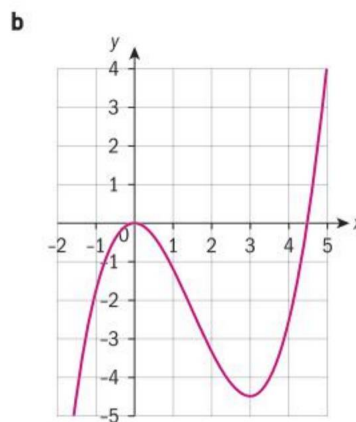**12** **P1:** A function is given by $f(x) = 128x - 15$, $-3 < x < 15$.

a Determine the value of $f\left(\dfrac{3}{2}\right)$.
(2 marks)

b Determine the range of the function f.
(4 marks)

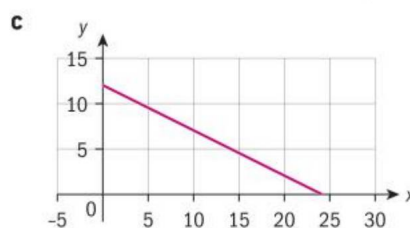**c** Determine the value of a such that $f(a) = 1162.6$ (2 marks)

13 **P1:** State **i** the domain, and **ii** the range for each of the following functions.

a

(2 marks)

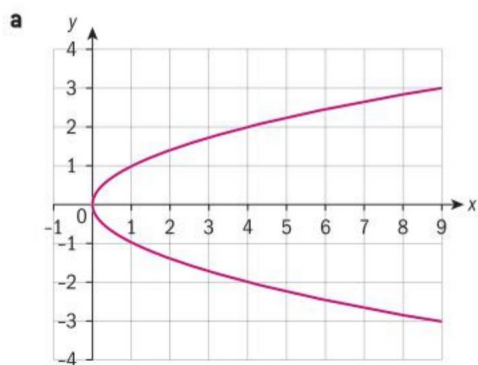**b**

(2 marks)

c

(2 marks)

d
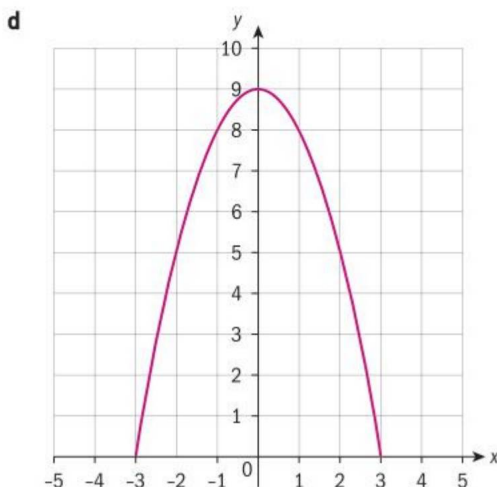
(2 marks)

14 P2: A function $f(x)$ is defined as $f(x) = 3x - 10$

a Given that the range of $f(x)$ is $5 < f(x) < 50$, find the domain of $f(x)$. (4 marks)

b Find $ff(10)$. (3 marks)

c Find the inverse function, $f^{-1}(x)$. (2 marks)

d State the range of the inverse function. (2 marks)

15 P1: State which of the following graphs represent functions, giving reasons for your answers.

a

(2 marks)

b

(2 marks)

c

(2 marks)

d
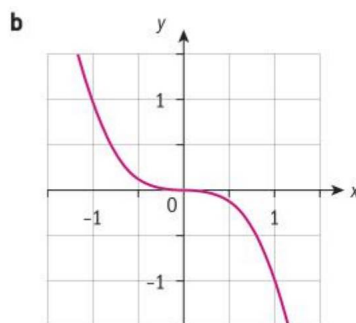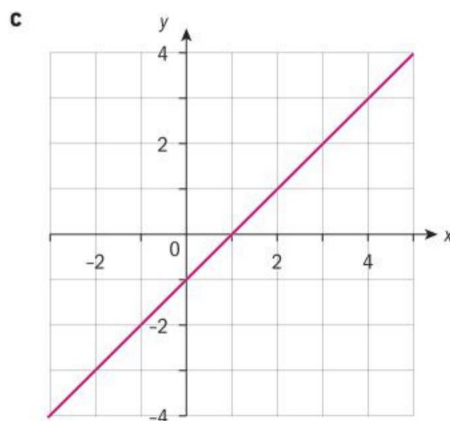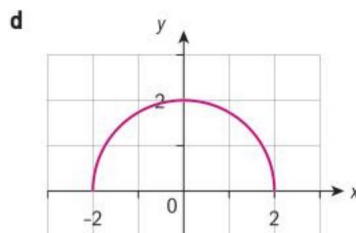
(2 marks)

16 P1: Consider the function $f(x) = \dfrac{k}{x-1} + 1$, $x > 1$, $x \in \mathbb{R}$, $k \in \mathbb{R}$.

a Show that $f(x)$ is a self-inverse function. (3 marks)

b State the range of f. (2 marks)

c Sketch the graph of $y = f(x)$. (2 marks)

17 P1: Consider the function $f(x) = \dfrac{1-2x}{3x+6}$, $x \neq -2$, $x \in \mathbb{R}$.

a State the range of f. (1 mark)

b Find the inverse function $f^{-1}(x)$. (3 marks)

c State the domain and the range of $f^{-1}(x)$. (2 marks)

18 P1: Consider the functions $f(x) = x^2$, $x \in \mathbb{R}$; and $g(x) = 2x - 1$, $x \in \mathbb{R}$.

a Solve the equation $f(x) = g(x)$. (3 marks)

b Solve the equation $fg(x) = gf(x)$. (5 marks)

19 P2: Katie organises a party for her work colleagues. She has a maximum budget of $1000.

The cost to rent a local hall is $430 for the evening.

She also has to budget for food, which will cost approximately $14.50 per person.

a Write down a formula connecting the total cost of the party (C) with the number of people attending the party (p). (2 marks)

b Explain why $C = f(p)$ is a function. (1 mark)

c Derive an expression for p in terms of C. (2 marks)

d Hence, calculate the greatest number of people Katie is able to invite. (2 marks)

e Given that only 16 people attend the party, calculate how much each guest should be charged so that Katie covers her costs. (3 marks)

20 P1: The function $h(x)$ is defined as $h(x) = \dfrac{x}{3} + 2$, $x \geq 0$, $x \in \mathbb{R}$.

a State the range of $h(x)$. (1 mark)

b Derive an expression for the inverse function, $h^{-1}(x)$. (3 marks)

c Find an expression for $hh(x)$ in the form $hh(x) = ax + b$, where a and b are constants. (3 marks)

d Solve the equation $h(x) = h^{-1}(x)$. (2 marks)

e Explain why the equation $h(x) = h^{-1}(x)$ has the same solutions as the equation $h(x) = x$. (1 mark)

21 P1: The function $p(x)$ is defined by $p(x) = x^2 + 4x - 11$, $x \in \mathbb{R}$.

Given that $p(x) = fgh(x)$ and $f(x) \neq x$, $g(x) \neq x$, $h(x) \neq x$, find possible functions for $f(x)$, $g(x)$, and $h(x)$. (5 marks)

22 P1: Consider the functions $f(x) = x^2 - 4$, $g(x) = \dfrac{1}{x+1}$, $h(x) = 2^x$, $x \in \mathbb{R}$.

a Find the range of $f(x)$. (1 mark)

b Find the range of $g(x)$. (1 mark)

c Find the range of $h(x)$. (1 mark)

d Find an expression for $gf(x)$. (2 marks)

e Solve the equation $gf(x) = 9$. (2 marks)

f Solve the inequality $gh(x) > \dfrac{1}{17}$. (5 marks)

23 P1: Consider the function $p(x) = x^3$, $-2 \leq x \leq 2$, $x \in \mathbb{R}$.

a Find the range of $p(x)$. (2 marks)

b Find an expression for the inverse function $p^{-1}(x)$. (2 marks)

c Find all the solutions to the equation $p(x) = p^{-1}(x)$. (2 marks)

d Sketch the graphs of $y = p(x)$ and $y = p^{-1}(x)$ on the same axes. (2 marks)

24 P1: a Show that $r(x) = \dfrac{3x + 5}{4x - 3}$ $x \in \mathbb{R}$, $x \neq \dfrac{3}{4}$ is a self-inverse function. (3 marks)

b Hence determine the value of $r(5)$. (2 marks)

25 P1: The function $f(x)$ is one-to-one and defined such that $f(x) = x^2 - 6x + 13$, $x \geq k$, $x \in \mathbb{R}$, $k \in \mathbb{R}$.

a Find the least possible value for k. (3 marks)

b Find an expression for the inverse function $f^{-1}(x)$. (3 marks)

c State the domain and the range of $f^{-1}(x)$. (2 marks)

26 P1: Given that $f(x) = x - 3$ and $gf(x) = 2x^2 + 18$, derive an expression for the function $g(x)$. (4 marks)