

## Chapter review

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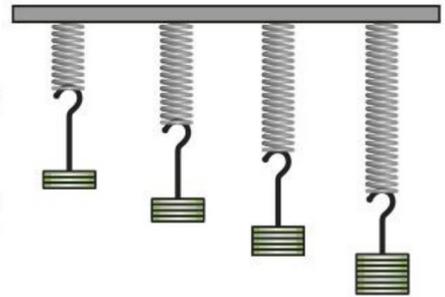


- For each of the following sequences,
  - Identify whether the sequence is arithmetic, geometric or neither.
  - If it is arithmetic or geometric, find an expression for  $u_n$ .
  - If it is arithmetic or geometric, find the indicated term.
  - If it is arithmetic or geometric, find the indicated sum.
    - $3, 6, 18, \dots u_8, S_{12}$
    - $-16, -14, -12, \dots u_{10}, S_8$
    - $2000, 1000, 500, \dots u_9, S_7$
    - $\sum_{n=1}^{\infty} 3 \times 2^{n-1}, u_5, S_{10}$
  - The consecutive multiples of 5 greater than 104,  $u_7, S_9$
- In an arithmetic sequence,  $u_6 = -5$  and  $u_9 = -20$ . Find  $S_{20}$ .
- Write down the first five terms for the recursive sequence  $u_n = -2u_{n-1} + 3$  with  $u_1 = -4$ .
- For the geometric series  $0.5 - 0.1 + 0.02 \dots$   $S_n = 0.416$ . Find the number of terms in the series.
- For a geometric progression,  $u_3 = 4.5$  and  $u_7 = 22.78125$ . Find the value of the common ratio and the first term.
- Which of the following sequences has an infinite sum? Justify your choice and find that sum.
 

A  $\frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \dots$     B  $0.06, 0.12, 0.24, \dots$
- How many terms are in the sequence  $4, 7, 10, \dots, 61$ ?
- In a geometric sequence, the fourth term is 8 times the first term. If the sum of the first 10 terms is 765, find the 10th term of the sequence.
- Three consecutive terms of a geometric sequence are  $x - 3, 6$  and  $x + 2$ . Find all possible values of  $x$ .
- A tank contains 55 litres of water. Water flows out at a rate of 7% per minute.
  - Write a sequence that represents the volume left in the tank after 1 minute, 2 minutes, 3 minutes, etc.
  - What kind of sequence did you write? Justify your answer.
  - How much water is left in the tank after 10 minutes?

- How much water has been drained after 15 minutes?
- How long will it take to drain the tank?

- 11 Marie-Jeanne is experimenting with weights of 100 g attached to a spring. She records the weight and length of the spring after attaching the weight.



Number of 100 g weights attached to the spring	1	2	3	4
Length of spring [cm]	45	49	53	57

- Write a general formula that represents the length of the spring if  $n$  weights are attached.
  - Calculate the supposed length of a spring if a 1 kg weight is attached.
  - Explain any limitations of Marie-Jeanne's experiment.
  - If the spring stretches to 101 cm, what is the total weight that was attached to the spring?
- Kostas gets a four-year bank loan to buy a new car that is priced at €20 987. After the four years, Kostas will have paid the bank a total of €22 960. What annual interest rate did the bank give him if the interest was compounded monthly?
  - The first seven numbers in row 14 of Pascal's triangle are 1, 13, 78, 286, 715, 1287, 1716.
    - Complete the row and explain your strategy.
    - Explain how you can use your answer in part a to find the 15th row. State the terms in the 15th row.
  - Using the binomial theorem, expand  $(3x - y)^6$ .
  - Find the coefficient of the term in  $x^2$  in the expansion of  $\left(\frac{3}{x^2} - 4x^4\right)^8$ .
  - In the binomial expansion of  $\left(\frac{2}{x^2} - 5x^3\right)^n$ , the sixth term contains  $x^{25}$ . Solve for  $n$ .

- 17 a Find the term in  $x^5$  in the expansion of  $(x-3)^9$ .  
 b Hence, find the term in  $x^6$  in the expansion of  $-2x(x-3)^9$ .

- 18 In the expansion of  $\left(\frac{x^3}{3} + \frac{k}{x}\right)^{12}$ , the constant is  $\frac{112\,640}{27}$ . Find the value of  $k$ .

- 19 Prove that:

$$(2x-1)(x-3) - 3(x-4)^2 = -x^2 - 31x + 51.$$

- 20 a Prove that  $\frac{x^2-x-6}{x+4} \cdot \frac{x^2-16}{x^2+2} = \frac{x^2-7x+12}{x}$ .  
 b For what values of  $x$  does this mathematical statement **not** hold true?

## Exam-style questions

- 21 P1: a Find the binomial expansion of  $\left(1 - \frac{x}{4}\right)^5$  in ascending powers of  $x$ . (3 marks)

- b Using the first three terms from the above expansion, find an approximation for  $0.975^5$ . (3 marks)



- 22 P1: The 15th term of an arithmetic series is 143 and the 31st term is 183.

- a Find the first term and the common difference. (5 marks)  
 b Find the 100th term of the series. (2 marks)

- 23 P2: Angelina deposits \$3000 in a savings account on 1 January 2019, earning compound interest of 1.5% per year.

- a Calculate how much interest (to the nearest dollar) Angelina would earn after 10 years if she leaves the money alone. (3 marks)  
 b In addition to the \$3000 deposited on January 1st 2019, Angelina deposits a further amount of \$1200 into the same account on an annual basis, beginning on 1st January 2020. Calculate the total amount of money in her account at the start of January 2030 (before she has deposited her money for that year). (4 marks)

- 24 P2: Brad deposits \$5500 in a savings account which earns 2.75% compound interest per year.

- a Determine how much Brad's investment will be worth after 4 years. (3 marks)

- b Calculate, to the nearest year, how long Brad must wait for the value of the investment to reach \$12 000. (5 marks)

- 25 P2: Find the coefficient of the term in  $x^5$  in the binomial expansion of  $(3+x)(4+2x)^8$ . (4 marks)

- 26 P1: The coefficient of  $x^2$  in the binomial expansion of  $(1+3x)^n$  is 495. Determine the value of  $n$ . (6 marks)

- 27 P2: Find the constant term in the expansion of  $\left(x^3 - \frac{2}{x}\right)^8$ . (4 marks)

- 28 P2: a Find the binomial expansion of  $\left(\frac{1}{2x} - x\right)^4$  in ascending powers of  $x$ . (3 marks)

- b Hence, or otherwise, find the term independent of  $x$  in the binomial expansion of  $(3-x)^3 \left(\frac{1}{2x} - x\right)^4$ . (4 marks)

- 29 P2: A convergent geometric series has sum to infinity of 120. Find the 6th term in the series, given that the common ratio is 0.2. (5 marks)

- 30 P1: The second term in a geometric series is 180 and the sixth term is  $\frac{20}{9}$ . Find the sum to infinity of the series. (7 marks)

- 31 P2: Find the value of  $\sum_{n=0}^{n=15} (1.6^n - 12n + 1)$ , giving your answer correct to 1 decimal place. (6 marks)

- 32 P1: A ball is dropped from a vertical height of 20 m. Following each bounce, it rebounds to a vertical height of  $\frac{5}{6}$  its previous height. Assuming that the ball continues to bounce indefinitely, show that the maximum distance it can travel is 220 m. (5 marks)

- 33 P1: Prove the binomial coefficient identity  $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ . (6 marks)

- 34 P2: Find the sum of all integers between 500 and 1400 (inclusive) that are not divisible by 7. (7 marks)